RPN64

MAGNETIC CONVERTER DESIGN CONSIDERATIONS

by

DAVID C. DEPACKH

6 February 1968

In Radiation Project Progress Report No. 2 certain preliminary estimates were made connected with the requirements on the converter field and power supply. Some of these will be amplified here. In particular further attention will be given to estimating the phase space of the linac beam and the dependence of the stored energy in the magnetic field on this.

The field must be shaped so that it can trap injected particles. This means that outside the region of radial focusing there must be a region of radial defocusing and an unstable equilibrium in between. With respect to radial motion this state of affairs can be represented by a phase space diagram (Figure 1), which shows the separatrix between stable and unstable orbits. In a time-independent field the incoming orbit must lie on the separatrix for trapping, which then occurs precisely at the saddle-point (unstable equilibrium) orbit. In the diagram the separatrix is drawn for one particular field and particle energy, which, for the given field geometry, determine the positions of the stable and unstable equilibrium orbits. As the field increases in time, the equilibrium orbit positions move to more widely separated radii and the phase space area of the separatrix increases, which therefore suggests that particle trapping may

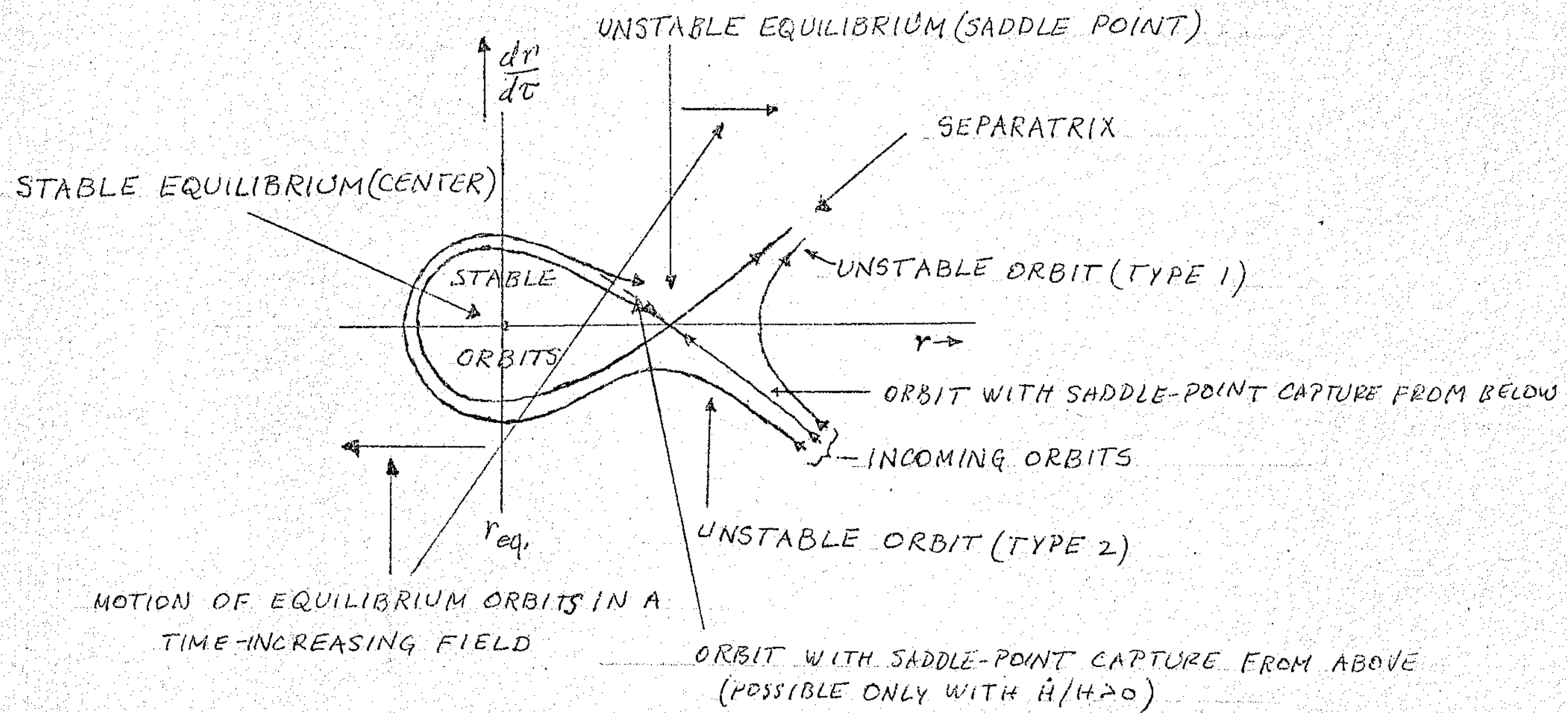


FIGURE 1. PILLBOX ORBITS

occur for initially unstable orbits indicated as type 2 in the figure if the separatrix expands far enough during the trajectory to capture the orbit. Computer calculations show that this is in fact the case. In the appendix a qualitative argument will be advanced to indicate about how much the original orbit may be displaced below the separatrix and still allow capture in a time-increasing field. Results given in the appendix according to these calculations indicate somewhat more stringent requirements on the linac beam than appeared in the original estimates, although the final results require the computer program.

The condition $n < 1$ ($n = -r\partial H/H\partial r$) in some suitable average sense is necessary but not sufficient for orbit stability. The orbits are known to be vertically unstable when $n < 0$, a condition which corresponds to field lines which diverge with increasing z. It is therefore not desirable to allow the radial oscillations to have too large an amplitude if, as seems desirable for stored-energy reasons, the n-value becomes negative within some particular radius. Fields for which the lines converge with increasing z at all radii store much energy in large-z regions where it is of no use in containing the electrons. It is thus suggested that to minimize the energy stored in the field one should limit the range of orbits so that the n=0 point is reached at some definite radius. If one assumes that the field is nearly constant in time during the radiation interval, then the 1-Gev electrons in a 1-MG field will radiate half their energy in a containment time of ~ 200 msec., which means that in this time the orbit radius will shrink by a factor somewhat greater than 2 (since the field is increasing inward). Thus if the ratio

$r(n=1)/r(n=0)$ is less than this factor, the orbits will become vertically unstable at the end of their containment and will leave through the ends of the converter. This kind of behavior can be of considerable benefit in getting rid of the spent electrons in the system, but it sets a lower limit on the stored energy.

The final coil design requires the computer, but a feel for the conductor configuration can be arrived at by considering single-filament circular coils. For determination of the field it is convenient to use the Ψ-function in axial symmetry, where $\Psi \equiv rA_\theta$. For the calculation we consider a fixed coil radius of 1 cm. We then have[1]

$$\Psi/r^{\frac{1}{2}} = 2ik^3\, C(k^2),$$

where[2] $C \equiv k^{-4}[(2-k^2)K-2E]$, and K and E are complete elliptic integrals of modulus k, and where

$$k^2 \equiv 4r/[(1+r)^2 + z^2].$$

Surfaces of constant Ψ correspond to field lines, so that conductors can be placed at them (i.e., they are analogous to electrical equipotentials). To find the fields we have $H_z = \partial\Psi/r\partial r$; $H_r = -\partial\Psi/r\partial z$; the magnetic flux through any surface of constant Ψ is $2\pi\Psi$. Figure 2 shows H_z/i and n for z = 0.5 (Helmholtz configuration) and z = 0.4. In the latter case one notes that $r(n=1)/r(n=0)$ is $0.77/0.41 = 1.87$, whereas we have $H(n=1)/H(n=0) = 0.7$, so that the orbit shrinkage under 50% energy loss is $2/0.7 = 2.83$, and $z = 0.4$ is therefore too small. It seems clear that one will not do much better than the Helmholtz values, so that the chief advantage in making z slightly less than 0.5 is to create an escape path for spent electrons.

It is advantageous to plot equi-Ψ surfaces since this allows calculation of the total contained flux and thus of the inductance $2\pi\Psi/i$. In particular, the more space is filled with conductor, the greater the stored energy reduction. This means making the current-carrying conductors as large as possible and perhaps even using a central conductor on an equi-Ψ surface so as to reduce the flux contained in the active region. The stored-energy advantage of such a conductor must be weighed against the undesirable interactions it may have with the escaping electrons. The geometric scale can be roughly set by recalling that 1 Gev, 10^6 gauss $\rightarrow$ 3.4-cm radius, while we find that for n=1, $r \sim 0.75$ in our 1-cm-radius coil, so that the scale factor is $3.4/0.75 = 4.53$. The current for z=0.5 is then $10^6/(3.0/4.53) = 1.51 \times 10^6$ emu, while for z=0.4 it is $10^6/(3.6/4.53) = 1.26 \times 10^6$ emu.

An approximate graphical estimate of the z=0.5 separatrix (figure 3; a conductor placed on this surface just closes the gap between the upper and lower coils) indicates the maximum Ψ-value which admits communication of the interior of the configuration with the outside along the median plane; this is at r=1.3, Ψ=2.05. Thus the total flux (for one conductor) is $2.05 \times 2\pi = 12.9$,

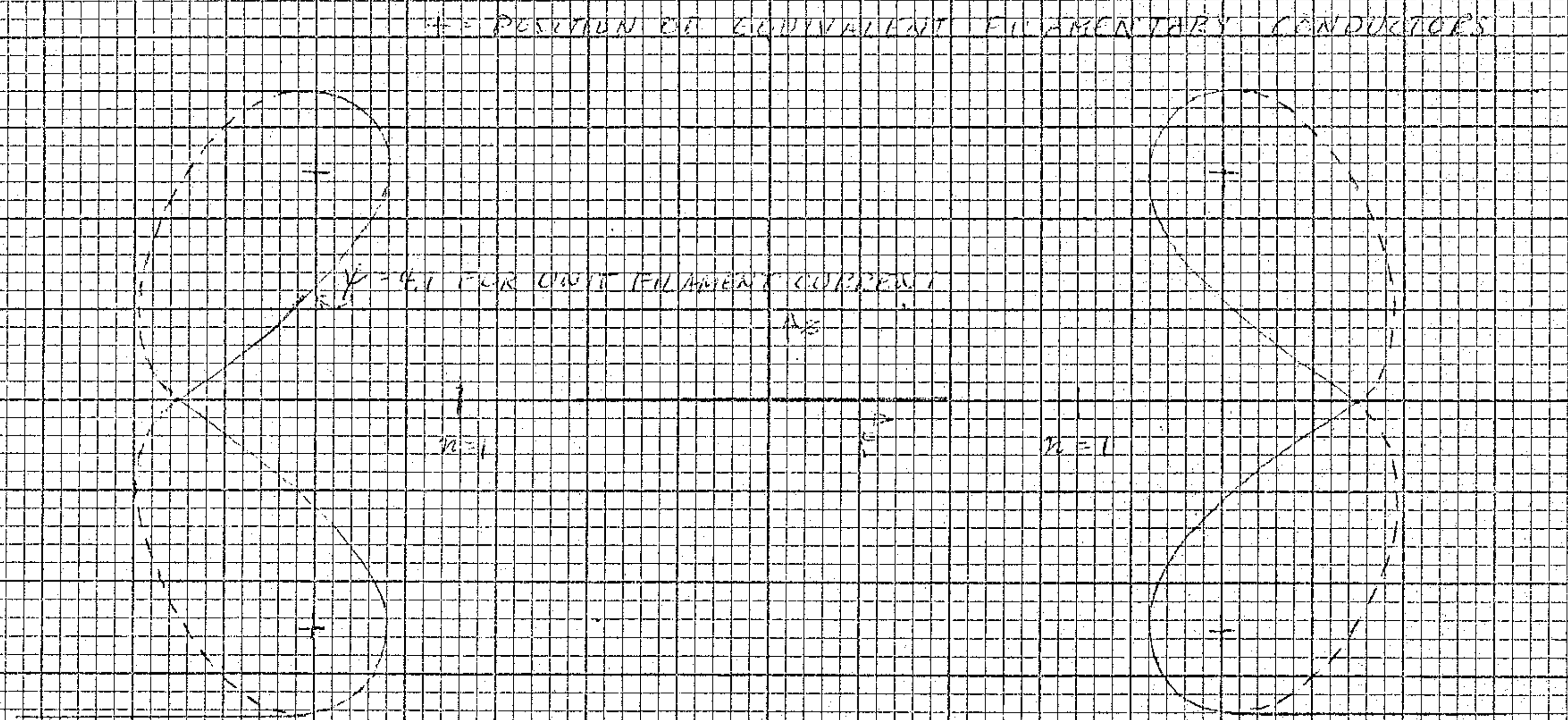


FIGURE 3 - POSITIONS OF CONDUCTORS ON SEPARATRIX FIELD LINES. CONDUCTOR CAN PRESUMABLY BE CONTINUED AT RADII GREATER THAN THE Z-MAXIMUM OF THE SEPARATRIX WITHOUT MATERIALLY AFFECTING THE INTERNAL FIELD. SIMILARLY, A SLOT FOR BEAM AND RADIATION TRANSMISSION CAN PROBABLY BE CUT OUTWARD FROM THE SADDLE POINT. HELMHOLTZ CONFIGURATION IS SHOWN

so that the inductance (for 1 emu) is 12.9 mh, or, with the scale factor of 4.53, the inductance for this configuration is 12.9 x 4.53 = 58.3 mh. A central conductor placed at r=0.3 lowers this by about 10%. The corresponding figures for z=0.4 are 69.2 mh and also about 10% inductance reduction due to a central conductor. The stored energies for the two cases (without central conductor) at 1 MG at n=1 are 6.64 MJ and 5.49 MJ.

It should be recalled here that the figures 1 Gev and 1 MG are primarily for reference. These stored-energy numbers certainly offer formidable difficulties and suggest that compromises in the particle energy may be needed. In a configuration of fixed shape the stored energy is proportional to $R^3H^2\propto\gamma^3/H$, where R is a linear dimension and the particles are relativistic. This dependence can be seen graphically in figure 4, showing lines of constant stored energy based on 6 MJ for 1 Gev, 1 MG. It is suggested that a reduction in beam energy, with a corresponding increase in current, may be desirable from this point of view, since it has been shown (Radiation Project Internal Report No. 3) that the linac power requirements are not strongly dependent on current.

The computer program which solves the Ψ-equation

$$\partial^2\Psi/\partial z^2 + r\partial/\partial r(\partial\Psi/r\partial r) = 0$$

does not have the conductor current as a primary datum; it must be determined instead by the integration

$$i = (1/4\pi)\oint \underline{H}\cdot d\,\underline{\ell}$$

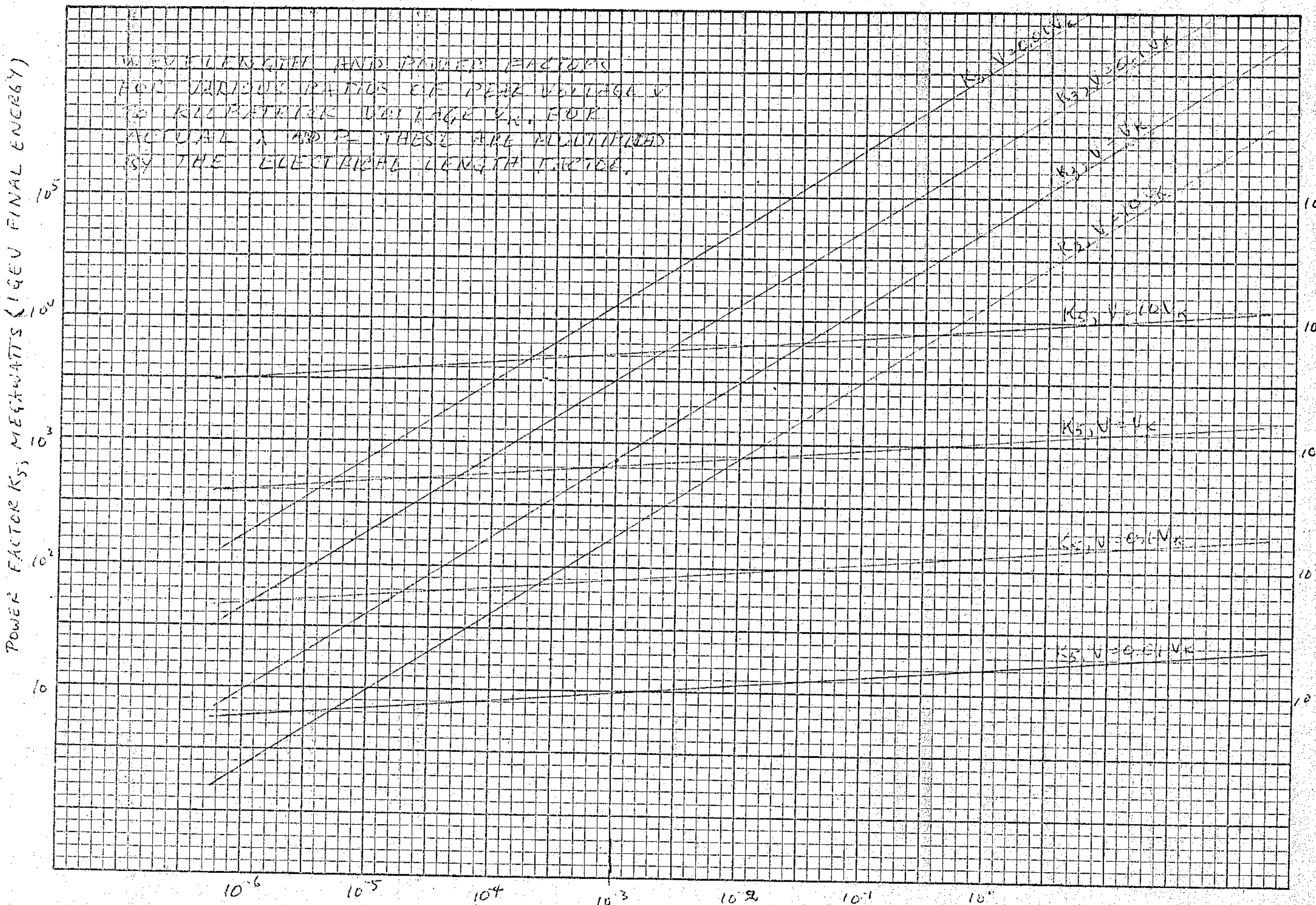

WAVE LENGTH AND POWER FACTORS FOR VARIOUS RATIOS OF PEAK VOLTAGE V TO KLYSTRON VOLTAGE Vk. FOR ACTUAL λ AND P THESE ARE MULTIPLIED BY THE ELECTRICAL LENGTH FACTOR.
K5, V=100Vk
K5, V=Vk
K5, V=0.1Vk
K5, V=0.01Vk
POWER FACTOR K5, MEGAWATTS (1 GEV FINAL ENERGY)
10
10²
10³
10⁴
10⁵
10⁻⁶
10⁻⁵
10⁻⁴
10⁻³
10⁻²
10⁻¹
10⁰
(k/h) it0, COULOMBS

FIGURE 2

around the conductors, where Ψ is conveniently taken as unity. The program then also provides the inductance $2\pi/i$ and the stored energy πi. H_z and n should be read out on the median plane. A program giving an accurate (intervals of 0.02) table of $\Psi(r,z)$ for a filamentary conductor 1 cm in radius and a current of 1 emu would be convenient for rough designs of coil shapes, since with such information equi-Ψ surfaces could easily be sketched.

REFERENCES

1. Zworykin and Morton, Electron Optics and the Electron Microscope, John Wiley and Sons, 1945, p. 472.

2. Jahnke and Emde, Tables of Functions, Dover, 1945, p. 73, p. 82.

APPENDIX

For the equation of single-particle radial motion (vertical motion and radiation are disregarded) we use the simplified equation

$$\ddot{r} = c^2/r - (e/\gamma m)H_z .$$

We write $H_z \equiv Th(r/r_o)$, where r_o is the scale of the field and T is a function of time only. From adiabatic arguments we take it that $\gamma \propto T^{2/3}$. Thus if $\rho \equiv r/r_o$, $d\tau \equiv cdt/r_o$, $\lambda \equiv er_o T/mc^2$, prime $\equiv d/d\tau$, then

$$d\rho'/d\tau = 1/\rho \quad -\lambda h(\rho),$$
$$d\rho/d\tau = \rho' .$$

Thus

$$\tfrac{1}{2}\rho'^2 = \log \rho \quad -\lambda g(\rho) + C, \quad g \equiv \int_0^\rho h(x)\, dx .$$

The center and saddle point locations are given by the solutions of

$$\rho h(\rho) = 1/\lambda .$$

In the interesting region the left-hand side is a positive function of ρ with a single maximum corresponding to $n=1$. For λ large enough there are two roots, the smaller of which corresponds to the center (stable equilibrium) and the larger (denoted by ρ_s) to the saddle point (unstable equilibrium).

For a given λ the value of the arbitrary constant C determines the particular orbit (depending on initial conditions). The equation of the separatrix is

$$\tfrac{1}{2}\rho'^2 = \log(\rho/\rho_s) = \lambda[g(\rho) - g(\rho_s)].$$

To estimate the range of trapping, write

$$\tfrac{1}{2}\Delta\rho'^2 = \int (1/\rho - \lambda h)d\rho,$$

where the in tegral is along the phase-space orbit. If both ends of the path are at the same value of ρ, say ρ_s, then

$$\tfrac{1}{2}\Delta\rho'^2 = -\int \lambda h d\rho = -\left(\int_{\rho_s}^{\rho_{min}} \lambda h d\rho + \int_{\rho_{min}}^{\rho_s} \lambda h d\rho\right) \equiv -(\bar{\lambda}_2 - \bar{\lambda}_1)\Delta g,$$

where $\bar{\lambda}_1$, $\bar{\lambda}_2$ are appropriate averages of λ over the intervals (ρ_s, ρ_{min}) and (ρ_{min}, ρ_s) and $\Delta g = g(\rho_s) - g(\rho_{min})$. Then one can write

$$\tfrac{1}{2}\Delta\rho'^2 \cong -\lambda' \overline{\Delta\tau}\, \Delta g,$$

where $\overline{\Delta\tau}$ is an appropriate fraction of 1 turn, which must be found by more detailed calculation. Differential analyzer calculations for saddle-point capture (i.e., taking the initial ρ equal to ρ_s as is done here) give this fraction as about 0.3 in typical cases. So in the present units we take $\overline{\Delta\tau} = 0.3 \times 2\pi \sim 2$, giving for the range of trapped orbits

$$\left|\tfrac{1}{2}\Delta\rho'^2\right| = 2\lambda'\, \Delta g.$$

When, as is practically the case, injection occurs at a radius greater than ρ_s, we assume that $\rho'\Delta\rho'$ is given by this same quantity: thus in general

$$\left|\rho'\Delta\rho'\right| = 2\lambda' \Delta g.$$

To find the horizontal phase space at injection, refer to figure 5, from which we have

$$\Psi + \phi = \theta,$$
$$d\eta/dx = -\tan \Psi,$$
$$d\rho/d\tau = -\tan \phi;$$

τ is measured along the incoming trajectory in units of r_o with the velocity fixed at c. We suppose that Ψ is small; then

$$d\rho/d\tau \sim - d\eta/dx + \tan[\text{arc} \cos(1-\eta/\rho_i)],$$

where ρ_i is the injection radius, presumably the zero-field point. The capturable range of ρ' is given by

$$\rho_i' < \rho' < \rho_i' + 2\lambda'\Delta g/\rho_i',$$

where ρ_i' is the value of ρ' on the separatrix at $\rho=\rho_i$. Acceptable phase space boundaries are therefore

$$d\eta/dx = -\rho_i' + \tan[\text{arc} \cos(1-\eta/\rho_i)]$$

and

$$d\eta/dx = -\rho_i' + \tan[\text{arc} \cos(1-\eta/\rho_i)] - 2\lambda'\Delta g/\rho_i'.$$

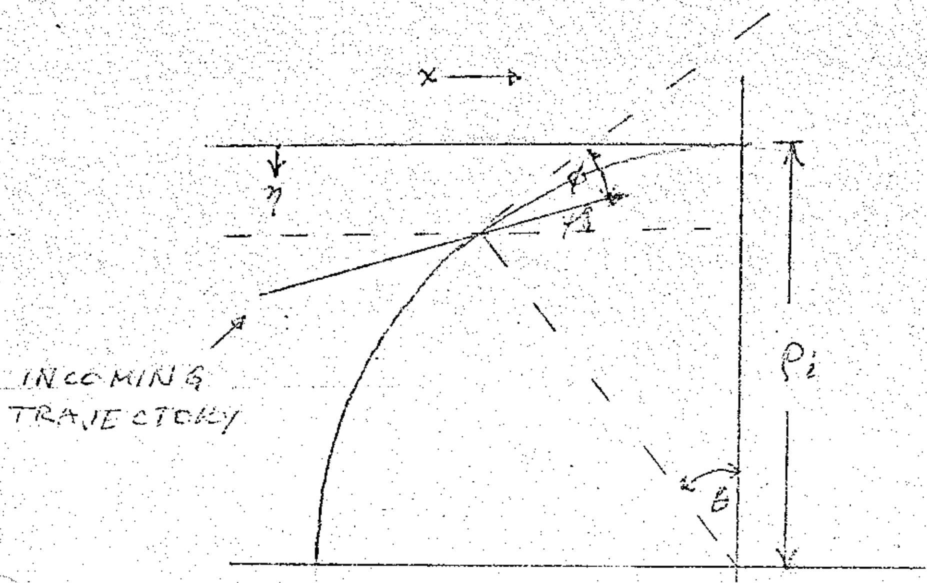


FIGURE 5: INJECTION GEOMETRY

The phase space is sketched roughly in figure 6. The height is about $2\lambda' \Delta g/\rho'$ and the width is about ρ_i; tilting of the injected phase space figure is needed for optimizing the fit, which implies that horizontal convergence of the entering beam is indicated. Optimally the required phase space is thus of the order of

$$2\rho_i \lambda' \Delta g/\rho_i'.$$

Reverting to the original, practical variables, we have

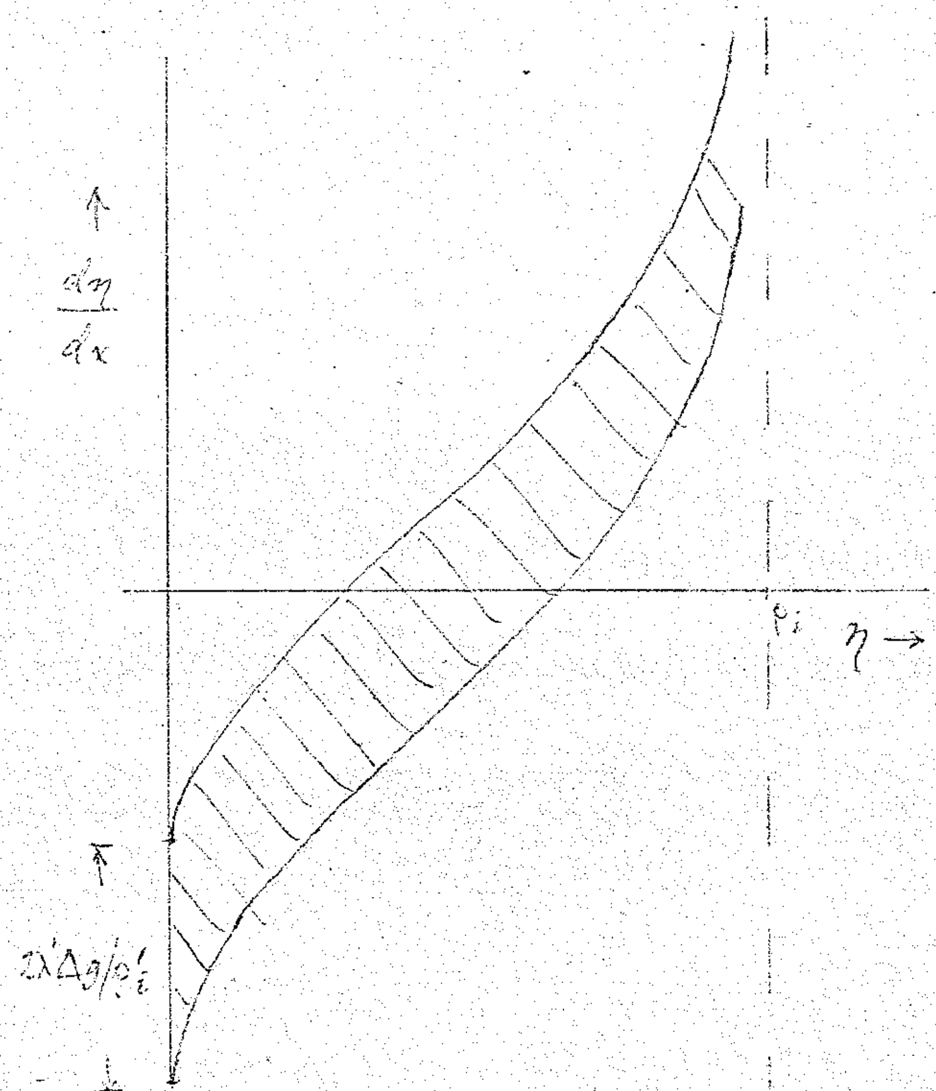
$$\rho' \Delta\rho' = (r_o/c)(\dot{H}/H)\Delta g,$$

$$\text{horizontal phase space} = (\rho' \Delta\rho')/\rho_i'.$$

The quantities Δg and ρ_i' must be found by detailed examination of the magnetic field shape. Typically such quantities are of unit magnitude.

To examine the required energy tolerance consider the time-independent situation, for which it can be seen that one can at fixed ρ define ρ'_{sep} as a function of $1/\lambda$, which plays the role of energy in these units. Thus any value of $1/\lambda$ up to the maximum, which is just accepted at the n=1 point, is allowed, but, since ρ' is a function of $1/\lambda$, these incoming trajectories must be variously deflected as the beam is introduced. Accordingly an energy-sensitive angular deflector is needed at the injection point. Generally one will expect perhaps a 10%

FIGURE 6: ACCEPTABLE HORIZONTAL PHASE SPACE



spread in linac energy, and it is assumed that a magnetic deflector capable of sufficient dispersion ($\equiv d\rho'/d(1/\lambda)$) can be designed. Thus the entrance optics might be as shown in figure 7. There are naturally substantial and detailed electron-optics problems involved in this design, which it is not the intention to go into here.

FIGURE 7: SCHEMATIC OF INJECTOR OPTICS